

Since  $u(0) = 0$ ,  $\dot{u}(0) = 0$  and  $\ddot{u}(0) = 0$  we have  $q(0) = 0$  et  $q'(0) = 0$ .  
 Thus, if  $q$  is a degree  $n$  polynomial, then  $q(u) = u^2 p(u)$  where  $p$  is a degree  $n - 2$  polynomial, and  $\sqrt{q(u)}$  is lipschitz in the neighborhood of 0.  
 From this it follows that the initial value problem associated to finding  $u(t)$

$$\begin{cases} \dot{u}(t) &= \sqrt{q(u(t))} \text{ for } t \geq 0 \\ u(0) &= 0 \end{cases}$$

has a unique solution, being of course the solution  $u(t) = 0$ , i.e. the motion gets stuck at standstill.

Furthermore,  $q''(u(t))\dot{u}(t) = 2\ddot{u}(t)$  and hence, if  $\ddot{u}(0) > 0$ , then  $\lim_{u \rightarrow 0^+} q''(u) = \infty$  and the function  $q$  cannot be well approximated by a polynomial!

Indeed, the time-optimal solution right at the beginning of the trajectory starting from standstill must feature a maximum jerk phase. If  $u = \frac{1}{6} j_{ps} t^3$  then  $t = \left(\frac{6u}{j_{ps}}\right)^{\frac{1}{3}}$  and  $q(u) = \frac{3}{2} 6^{\frac{1}{3}} j_{ps}^{\frac{2}{3}} u^{\frac{4}{3}}$ . Note the fractional power in the expression of  $q(u)$ .

The above explanations motivate our approach used at standstill.